Worksheet – Integration – "u du" Substitution (#3)

Integrate the following.

1) $\int \frac{4y}{\sqrt{2y^2+1}}\,dy$

2) $\int \frac{x^2+2x}{\sqrt{x^3+3x^2+1}}\,dx$

3) $\int \frac{4\sin x}{(1+\cos x)^2}\,dx$

4) $\int \sin(2x)\sqrt{2-\cos(2x)}\,dx$

5) $\int x\left(x^2+2\right)^{1/3}\,dx$

6) $\int \frac{10\sqrt{x}}{\left(1+x^{3/2}\right)^3}\,dx$

7) $\int \frac{\left(x^{1/3}+2\right)^4}{\sqrt[3]{x^2}}\,dx$

8) $\int \frac{1}{2}t\cos\left(4t^2\right)\,dt$

1) $\int \frac{4y}{\sqrt{2y^2+1}}\,dy$

$2y^2+1=u$

$4y\,dy=du$

$dy=\frac{du}{4y}$

$\int 4y(2y^2+1)^{-1/2}$

$\int \cancel{4y}\,(u)^{-1/2}\frac{du}{\cancel{4y}}$

$\int u^{-1/2}\,du$

$y=2u^{1/2}+C$

$y=2(2y^2+1)^{1/2}+C$

2) $\int \frac{x^2+2x}{\sqrt{x^3+3x^2+1}}\,dx$

$\int (x^2+2x)(x^3+3x^2+1)^{-1/2}\,dx$

$u=x^3+3x^2+1$

$du=3x^2+6x\,dx$

$\frac{du}{3x^2+6x}=dx$

$\int \cancel{(x^2+2x)}\cdot u^{-1/2}\frac{du}{3\cancel{(x^2+2x)}}$

$\int \frac{1}{3}\cdot u^{-1/2}\,du$

$y=\frac{2}{3}u^{1/2}+C$

$y=\frac{2}{3}(x^3+3x^2+1)^{1/2}+C$

3) $\int \frac{4\sin x}{(1+\cos x)^2}\,dx$

$\int 4\sin(x)\cdot(1+\cos(x))^{-2}$

$u = 1+\cos(x)$

$du = -\sin(x)\,dx$

$\frac{du}{-\sin(x)} = dx$

$\int 4\cdot\sin(x)\cdot(u)^{-2}\frac{du}{-\sin(x)}$

$\int -4u^{-2}\,du$

$y = 4u^{-1} + C$

$y = 4(1+\cos(x))^{-1} + C$

4) $\int \sin(2x)\sqrt{2-\cos(2x)}\,dx$

$u = 2-\cos(2x)$

$du = 2\sin(2x)\,dx$

$\frac{du}{2\sin(2x)} = dx$

$\int \sin(2x)(u)^{1/2}\frac{du}{2\sin(2x)}$

$\int \frac{1}{2}(u)^{1/2}\,du$

$y = \frac{1}{3}u^{3/2} + C$

$y = \frac{1}{3}(2-\cos(2x))^{3/2} + C$

5) $\int x(x^2+2)^{1/3}\,dx$

$u = x^2+2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\int x(u)^{1/3}\frac{du}{2x}$

$\int \frac{1}{2}u^{1/3}\,du$

$y = \frac{3}{8}u^{4/3}+C$

$y = \frac{3}{8}(x^2+2)^{4/3}+C$

6) $\int \frac{10\sqrt{x}}{(1+x^{3/2})^3}\,dx$

$u = 1+x^{3/2}$

$du = \frac{3}{2}x^{1/2}\,dx$

$\frac{du}{\frac{3}{2}x^{1/2}} = dx$

$\int 10x^{1/2}(u)^{-3}\frac{du}{\frac{3}{2}x^{1/2}}$

$\int \frac{20}{3}u^{-3}\,du$

$y = -\frac{10}{3}u^{-2}+C$

$y = -\frac{10}{3}(1+x^{3/2})^{-2}+C$

7) $\int \frac{\left(x^{1/3}+2\right)^4}{\sqrt[3]{x^2}}\,dx$

$u = x^{1/3} + 2$

$du = 1/3\, x^{-2/3}\,dx$

$\int (x^{1/3}+2)^4 \cdot x^{-2/3}\,dx$

$\frac{du}{\frac{1}{3}x^{-2/3}} = dx$

$\int u^4 \cdot x^{-2/3} \cdot \frac{du}{\frac{1}{3}x^{-2/3}}$

$\int 3u^4\,du$

$y = \frac{3}{5}u^5 + c$

$y = \frac{3}{5}(x^{1/3}+2)^5 + c$

8) $\int \frac{1}{2}t\cos\left(4t^2\right)dt$

$u = 4t^2$

$du = 8t\,dt$

$\frac{du}{8t} = dt$

$\int \frac{1}{2}t \cdot \cos(u) \cdot \frac{du}{8t}$

$\int \frac{1}{16} \cdot \cos(u)\,du$

$y = \frac{1}{16}\sin(u) + c$

$y = \frac{1}{16}\sin(4t^2) + c$